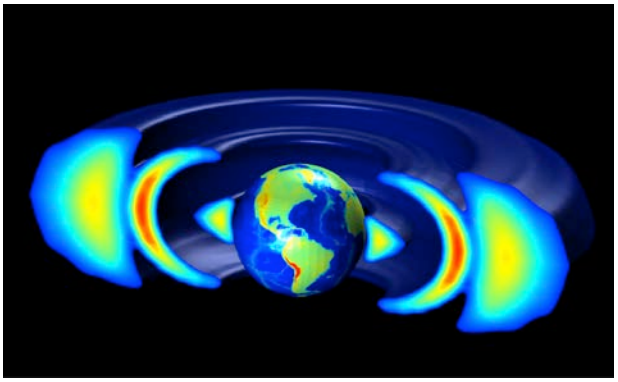


(Image credit: Yuri Shprits, Adam Kellerman, Dmitri Subbotin/UCLA)

The Van Allen Probes spacecraft travel in an elliptical orbit through the Van Allen belts. Soon after launch, they detected a third radiation belt shown by the middle crescent in the figure to the left.

In this problem, we predict when the spacecraft will encounter this third belt along their orbit, so that scientists can schedule observations of this new region.

The equation for the orbit of the spacecraft is given in Standard Form by

$$1 = \frac{x^2}{6.25} + \frac{y^2}{9.0}$$

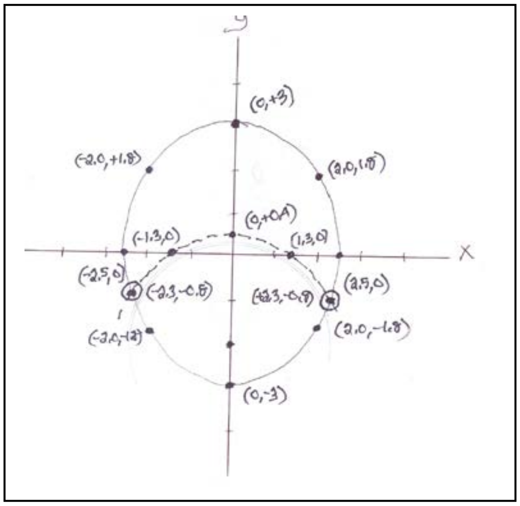
The equation for the location of the third Van Allen belt projected into the plane of the elliptical orbit and concentric with the orbit focus centered on Earth (0,-2) is given by

$$5.8 = x^2 + (y+2)^2$$

Problem 1 – On the same coordinate plane, graph these two functions and estimate the coordinates of the intersection points to the nearest tenth.

Problem 2 – Using only algebra, find the coordinates of all intersection points between the orbit and the new belt region to the nearest tenth.

Problem 1 – On the same coordinate plane, graph these two functions and estimate the coordinates of the intersection points to the nearest tenth. Answer: (-2.3, -0.9), (+2.3, -0.9).



Problem 2 – Using only algebra, find the coordinates of all intersection points between the orbit and the new belt region to the nearest tenth.

Answer: Expand out all terms and simplify

Equation of circle: $5.8 = x^2 + y^2 + 4y + 4.0$ so
$1.8 = x^2 + y^2 + 4y$

Equation of ellipse: $9.0 = 1.54x^2 + y^2$

Eliminate x^2 in equation of circle by using equation of ellipse solved for x^2.

$1.8 = \mathbf{(9.0 - y^2)/1.54} + y^2 + 4y$
$1.8 - 5.84 = -0.65y^2 + y^2 + 4y$
$4.0 = 0.35y^2 - 4y$

Solve the quadratic equation in y for its two roots: $0 = 0.35y^2 - 4y - 4.0$
A = +0.35, b = -4.0 c = -4.0 then

$Y = [4.0 +/- (16+5.6)^{1/2}]/0.70$ so Y1 = +12.3 Y2 = -0.86

Then from the equation for the ellipse we get the x coordinates:

Y1 = +12.3 : $X1 = +/- ((9.0 - (12.3)^2)/1.54)^{1/2}$ x1 = <u>imaginary solution</u>! (+/- 9.6 i)

Y2 = -0.86 : $X2 = +/- ((9.0 - (0.86)^2)/1.54)^{1/2}$ = +/- 2.30

Rounded to the nearest tenth we have +/- 2.8 so the real coordinates are
(+2.3, -0.9) and (-2.3, -0.9)