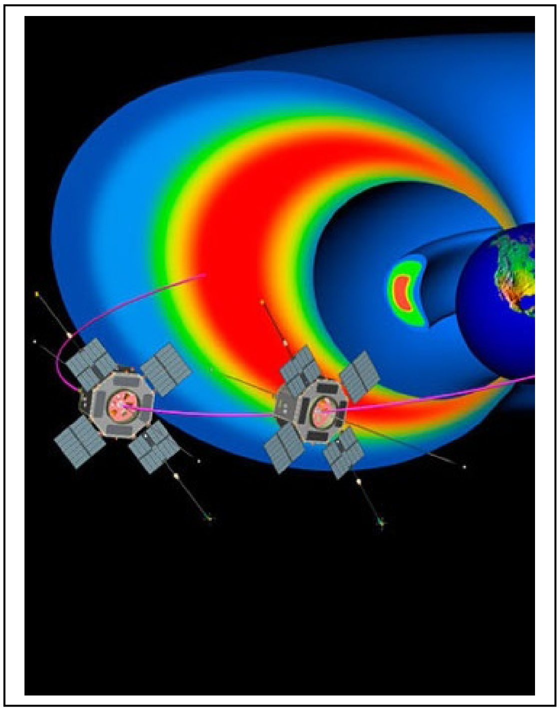
Modeling the Radiation Dose of the Van Allen Probes



The Van Allen Probes orbit Earth within the Van Allen belts about once every 9 hours. The radiation environment contains high energy electrons and protons that can damage satellite solar panels and delicate electronics without proper shielding.

Typical spacecraft used for scientific research will become unreliable after they have accumulated 1000 Grays of radiation, where 1 Gray = 1 joule of energy delivered to 1 kilogram of material. Usually this energy is harmlessly deposited into the spacecraft shielding. Some of it can, however, damage electronic circuits.

After a spacecraft accumulates about 1000 Grays of radiation, the wear on the satellite electronics can cause glitches, circuitry damage, and the satellite becomes unreliable.

An engineer wants to model the accumulation of radiation dosage as a satellite travels through the Van Allen belts in a 10-hour orbit. He wants to predict how many years of productive life the satellite will have given the shielding that it has before it reaches the 1000 Gray limit. There are two parts to this problem. The first is that he has to model the path taken by the spacecraft through the Van Allen belts. The second part is that he has to model how intense the radiation dose rate is in the Van Allen belts. Combining the two will give a model of the radiation dose rates the satellite will encounter at each point along its path.

Problem 1 – Suppose the radial distance between the center of Earth and the spacecraft can be modeled as a simple linear function during the time the shielded spacecraft is inside the Van Allen belts, and the radiation dose rate is modeled by a simple power-law function:

Path: $R(T) = 7000 + 3000T$ kilometers, where T is the elapsed time in hours.
Dose Rate: $D(R) = 60(R/25000)^2$ milliGrays/hour, where R is in kilometers.

What is the dose rate formula re-written so that the dose rate is a function of time D(T)?

Problem 2 – The integral of the dose rate formula D(T) with respect to time is the accumulated total dose.

A) Perform this integration for one 10-hour orbit of the spacecraft assuming that the total dose over the time interval T: [0h, 10h] is equal to twice the dose rate over the time interval T: [0h, 5h].

B) How many years will it take for the spacecraft total dose to equal 1000 Grays?

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Answer: Substitute the formula R(T) in the equation for D to get

$$D(T) = 60\left(\frac{7000+3000T}{25000}\right)^2 \quad \text{so} \quad D(T) = 60\left(\frac{7+3T}{25}\right)^2 \quad \text{milliGrays/hour}$$

Problem 2 – The [definite] integral of the dose rate formula D(T) with respect to time is the accumulated total dose.

A) Perform this integration for one 10-hour orbit of the spacecraft assuming that the total dose over the time interval T: [0h, 10h] is equal to twice the dose rate over the time interval T: [0h, 5h]. Answer:

$$\text{Total Dose} = 120\int_0^5 \left(\frac{7+3T}{25}\right)^2 dT$$

$$= \frac{120}{625}\int_0^5 (49+42T+9T^2)dT$$

$$= \frac{120}{625}\left(49T+21T^2+3T^3\right)_0^5$$

$$= \frac{120}{625}\left(49x5+21x25+3x125\right) \quad = \quad \text{220 milliGrays} = \textbf{0.22 Grays/orbit.}$$

B) How many years will it take for the spacecraft total dose to equal 1000 Grays?

Answer: The spacecraft accumulates 0.22 Grays every 10 hours, so in one year it accumulates

0.22 Grays/10 hours x (24 hours/1day)x(365 days/1 year) = 192 Grays/year,

then 1000 Grays/ (192 Gy/yr) = **5.2 years**.