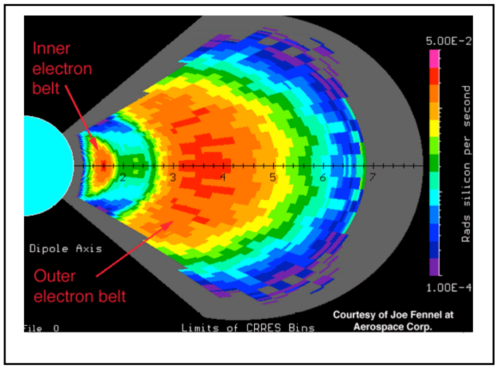
An Improved Model for Van Allen Belt Radiation Doses



Spacecraft engineers design spacecraft by considering the radiation environment, planned duration of the research program, and how much shielding is needed for the spacecraft to survive to the end of its mission.

In this problem we will work with a more realistic model for the orbit path and radiation belt dose rates, and perform a simple graphical integration by estimating areas under curves.

Problem 1 - The distance from the center of Earth to the spacecraft is given by the function

$$R(\theta) = 5.7 - \left[\frac{210}{100 - 55\cos\theta}\right]$$ in Earth radii units (where 1.0 = 6378 km)

$$T(\theta) = \frac{9}{2\pi}\left(\theta - 0.55\sin\theta\right)$$ hours

A better model of radiation dose rates into an unshielded silicon material G(R) in Grays/hour is given by the sixth-order function

$$G(R) = 0.136R^6 - 2.194R^5 + 13.89R^4 - 43.73R^3 + 71.78R^2 - 57.95R + 18.15 \text{ Gys/hour}$$

where R is the distance in multiples of Earth's radius. What is the function G(T)?

Problem 2 - For most realistic situations, it is almost impossible to find an analytic solution to the required integral to exactly determine the area under the curve, which in this case is the total integrated dose to the satellite in one orbit. It is common to use numerical integration techniques to construct an approximate solution, or to estimate the area under the curve by 'counting squares' graphically.

A) Graph the function G(T) over one complete orbit over the domain T: [0 , 9 hours].

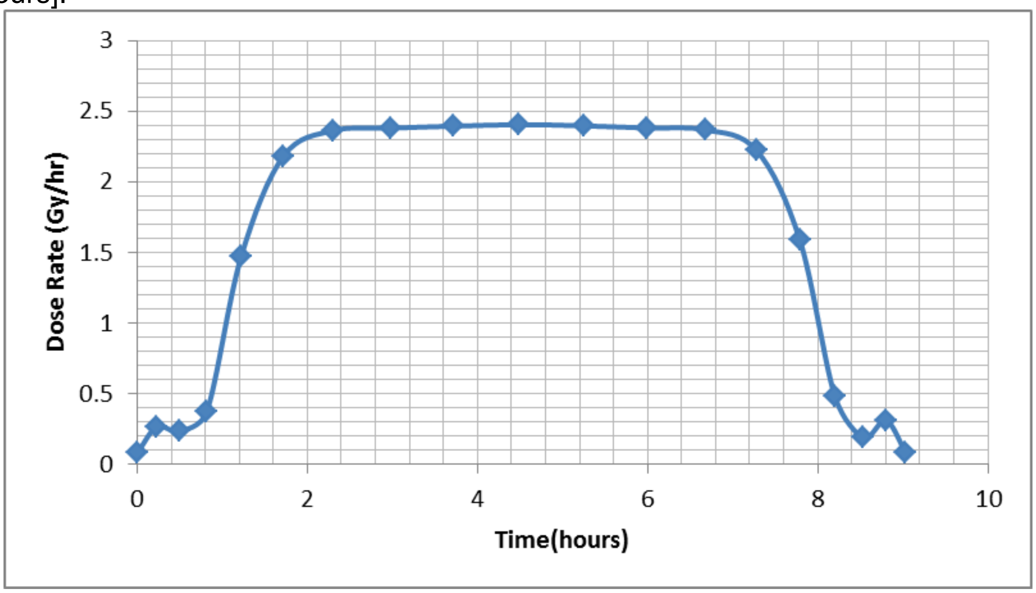
B) Estimate the geometric area under the curve using any method. What are the units of your estimated area rounded to 2 significant figure accuracy?

Problem 3 - A spacecraft will be at the end of its life after it has accumulated about 1000 Grays of radiation dose. A 1cm thickness of aluminum will reduce the radiation by 15 times. How many cm of shielding will be needed to reduce the total dose to 1000 Grays over 5 years?

Problem 1 - Answer. A student's first approach is to replace 'R' in the function G(R) with the function R(θ), and then replace θ by inverting the equation for T(θ) so that we have G(R(θ(T))). For realistic functions, this is a depressing and frustrating approach. The student should realize that a perfectly reasonable alternative is to define a function table where the columns are Θ, T(θ), R(θ), and G(R). A sample of such a table is shown here:

Θ in degrees	T: Time (hrs)	R in Re	G in Grays/hr
0	0.0	1.0	0.08
40	0.5	2.1	0.23
80	1.2	3.4	1.5
120	2.3	4.0	2.4
160	3.7	4.3	2.4
200	5.3	4.3	2.4
240	6.7	4.1	2.4
280	7.8	3.4	1.6
320	8.5	2.1	0.19

Problem 2 - A) Graph the dose rate function G(T) over one complete orbit over the domain T: [0 , 9 hours].



B) Answer: Graph: Total squares = 2x[½(2x4) + 4x4 + ½(4x20)+24x5.5)] = 384 squares
Table: 1 hr x (0.08+0.23+1.5+2.4+2.4+2.4+2.4+1.6+0.19) = **13.2** Grays/orbit
Area for 1 square = (0.4 hours) x (0.1 Grays/hr) = 0.04 Grays.
More accurate total dose is 0.04 x 384 squares= **15.4** Grays every 9 hour orbit.

Problem 3 - A spacecraft will be at the end of its life after it has accumulated about 1000 Grays of radiation dose. A 1cm thickness of aluminum will reduce the radiation by 15 times. How many cm of shielding will be needed to reduce the total dose to 1000 Grays over 5 years?
Answer: 15.4 Grays/10 hours x (8760 hours/1 year) x 5 years = 67500 Grays. Will need 67500/1000 = a factor of 68 reduction. 1 cm x (68/15) = **4.5 cm of shielding**