



NASA's Van Allen Probes satellites are in the shape of an octagon with a thickness of 84 centimeters between the front octagonal face and the back octagonal face. An engineer needs to determine the total surface area of this 'octagonal prism' in order to create a mathematical model of how fast the satellite is warming up and cooling down as it orbits Earth.

**Problem 1** – The figure on the right shows how the geometric area of an octagon can be broken up into rectangles, squares and triangles. What are the formulas for the areas of each of the squares, rectangles and triangles?

**Problem 2** – What is the formula for the total area of the octagonal face in terms of the measurements for  $a$  and  $b$ ?

**Problem 3** – What is the formula for the total surface area of the spacecraft if  $h$  is the distance between the top and bottom octagonal faces?

**Problem 4** - The engineer determines that  $a + 2b = 1.7$  meters and  $a = 0.7$  meters and  $h = 0.84$  meters. What is the total surface area to the nearest tenth of a square meter of A) one octagonal face? B) the entire satellite?

**Problem 1** – The figure on the right shows how the geometric area of an octagon can be broken up into rectangles, squares and triangles. What are the formulas for the areas of each of the squares, rectangles and triangles?

Answer:  $A(\text{square}) = a \times a = a^2$      $A(\text{rectangle}) = a \times b$      $A(\text{triangle}) = \frac{1}{2} b \times b$

**Problem 2** – What is the formula for the total area of the octagonal face in terms of the measurements for a and b?

Answer:  $A = 1 \times A(\text{square}) + 4 \times A(\text{rectangle}) + 4 \times A(\text{triangle})$   
 $A = a^2 + 4ab + 2b^2$

**Problem 3** – What is the formula for the total surface area of the spacecraft if h is the distance between the top and bottom octagonal faces?

Answer: There are two octagonal areas and 8 rectangular side faces each with an area of  $a \times h$ , so the total area of the spacecraft is

$$A = 2(a^2 + 4ab + 2b^2) + 8ah$$

**Problem 4** - The engineer determines that  $a + 2b = 1.7$  meters and  $a = 0.7$  meters and  $h = 0.85$  meters. What is the total surface area to the nearest tenth of a square meter of A) one octagonal face? B) the entire satellite?

Answer:  $a + 2b = 1.7$  meters and  $a = 0.7$  meters so  $b = 0.5$  meters

A)  $A = a^2 + 4ab + 2b^2$  so  $A = (0.7)^2 + 4(0.7)(0.5) + 2(0.5)^2 = 2.4 \text{ meter}^2$ .

B)  $A = 2(2.4 \text{ m}^2) + 8(0.7)(0.85) = 9.6 \text{ meters}^2$