The Van Allen belts were discovered in the late-1950s and resemble two donut-shaped clouds of protons (inner belt) and electrons (outer belt) with Earth at its center.

A donut is an example of a simple mathematical shape called a **torus** that is created by rotating a circle with a radius of $r$, through a circular path with a radius of $R$.

In terms of the variables $r$ and $R$, the formula for the volume of a torus is given by the rather scary-looking formula:

$$ V = 2\pi^2 R r^2 $$

**Problem 1** – What is the circumference of the circle with a radius of $R$?

**Problem 2** – What is the area of a circle with a radius of $r$?

**Problem 3** – If you dragged the area of the circle in Problem 2, along a distance equal to the circumference of the circle in Problem 2, what would be the formula for the volume that you swept out?

**Problem 4** – If the Van Allen belts can be approximated by a torus with $r = 16,000$ km, and $R = 26,000$ km, to two significant figures, what is the total volume of the Van Allen belts in cubic kilometers?

**Problem 5** – To two significant figures, how many spherical Earths can you fit in this volume if $r = 6378$ km?
**Problem 1** – What is the circumference of the circle with a radius of R?

Answer: \( C = 2 \pi R \)

**Problem 2** – What is the area of a circle with a radius of r?

Answer: \( A = \pi r^2 \)

**Problem 3** – If you dragged the area of the circle in Problem 2, along a distance equal to the circumference of the circle in Problem 2, what would be the formula for the volume that you swept out?

Answer: Volume = Area x distance

= \( (\pi r^2) \times (2 \pi R) \)

= \( 2 \pi^2 R r^2 \)

**Problem 4** – If the Van Allen belts can be approximated by a torus with \( r = 16,000 \) km, and \( R = 26,000 \) km, to two significant figures what is the total volume of the Van Allen belts in cubic kilometers?

Answer:
\[
\begin{align*}
\text{r} &= 16000 \text{ km} \times (1000 \text{ m/1km}) = 16,000,000 \text{ meters} \\
\text{R} &= 26000 \text{ km} \times (1000 \text{ m/1km}) = 26,000,000 \text{ meters} \\
\text{V} &= 2 (3.14)^2 (2.6x10^7) (1.6x10^7)^2 \\
&= 1.3 \times 10^{23} \text{ meters}^3
\end{align*}
\]

**Problem 5** – To two significant figures, how many spherical Earths can you fit in this volume if \( r = 6378 \) km?

Answer: \( V = \frac{4}{3} \pi r^3 \)

\[
\begin{align*}
\text{V} &= 1.33 (3.14) (6.378x10^6 \text{ m})^3 \\
&= 1.1 \times 10^{21} \text{ meters}^3
\end{align*}
\]

So \( 1.3 \times 10^{23} \text{ meters}^3 / 1.1 \times 10^{21} \text{ meters}^3 = 118 \) \( \text{ or } 120 \text{ Earths!} \)