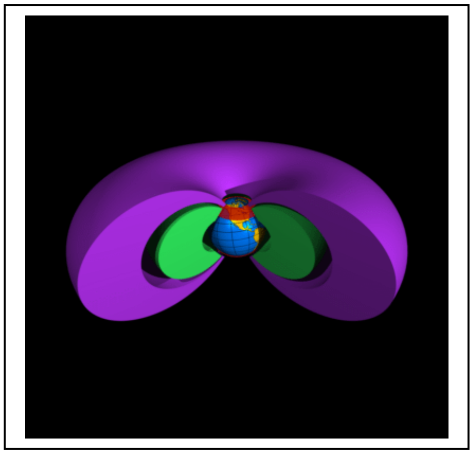
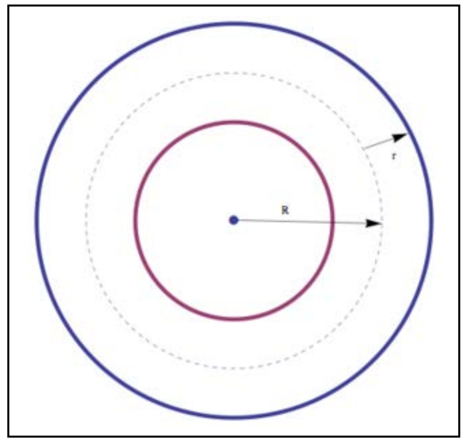
Exploring the Donut-shaped Van Allen Belts



The Van Allen belts were discovered in the late-1950s and resemble two donut-shaped clouds of protons (inner belt) and electrons (outer belt) with Earth at its center.

A donut is an example of a simple mathematical shape called a **torus** that is created by rotating a circle with a radius of r, through a circular path with a radius of R.



In terms of the variables r and R, the formula for the volume of a torus is given by the rather scary-looking formula:

$$V = 2\pi^2 R r^2$$

Problem 1 – What is the circumference of the circle with a radius of R?

Problem 2 – What is the area of a circle with a radius of r?

Problem 3 – If you dragged the area of the circle in Problem 2, along a distance equal to the circumference of the circle in Problem 2, what would be the formula for the volume that you swept out?

Problem 4 – If the Van Allen belts can be approximated by a torus with r = 16,000 km, and R = 26,000 km, to two significant figures, what is the total volume of the Van Allen belts in cubic kilometers?

Problem 5 – To two significant figures, how many spherical Earths can you fit in this volume if r = 6378 km?

Answer Key

Problem 1 – What is the circumference of the circle with a radius of R?

Answer: $C = 2 \pi R$

Problem 2 – What is the area of a circle with a radius of r?

Answer: $A = \pi r^2$

Problem 3 – If you dragged the area of the circle in Problem 2, along a distance equal to the circumference of the circle in Problem 2, what would be the formula for the volume that you swept out?

Answer: Volume = Area x distance

$$= (\pi r^2) \times (2 \pi R)$$

$$\mathbf{= 2 \pi^2 R r^2}$$

Problem 4 – If the Van Allen belts can be approximated by a torus with r = 16,000 km, and R = 26,000 km, to <u>two</u> significant figures what is the total volume of the Van Allen belts in cubic kilometers?

Answer:

r = 16000 km x (1000 m/1km) = 16,000,000 meters

R= 26000 km x (1000 m/1km) = 26,000,000 meters

$V = 2 (3.14)^2 (2.6 \times 10^7) (1.6 \times 10^7)^2$

$=$ **1.3×10^{23} meters^3**

Problem 5 – To <u>two</u> significant figures, how many spherical Earths can you fit in this volume if r = 6378 km?

Answer: $V = 4/3 \pi r^3$

$V = 1.33 (3.14) (6.378 \times 10^6 \text{ m})^3$

$V = 1.1 \times 10^{21}$ meters^3

So 1.3×10^{23} meters^3 / 1.1×10^{21} meters^3 = 118 or **120 Earths!**